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Class: B.A. Economics (Sem-2nd)
Paper: MIC-2
Topic: Chain rule of differentiation
Date: 15-04-2024

Chain Rule of Differentiation

If a function y = f(x) = g(u) and if u = h(x), then the chain rule for differentiation is defined as;

$$dy/dx = (dy/du) \times (du/dx)$$

This rule is majorly used in the method of substitution where we can perform differentiation of composite functions.

Let's have a look at the examples given below for better understanding of the chain rule differentiation of functions.

Example 1:

Differentiate $f(x) = (x^4 - 1)^{50}$

Solution:

Given,

$f(x) = (x^4 - 1)^{50}$

Let $g(x) = x^4 - 1$ and n = 50

$u(t) = t^{50}$

Thus, $t = g(x) = x^4 – 1$

$f(x) = u(g(x))$

According to chain rule,

$df/dx = (du/dt) \times (dt/dx)$

Here,

$du/dt = d/dt\ (t50) = 50t^{49}$

$dt/dx = d/dx\ g(x)$

$= d/dx\ (x^4 – 1)$

$= 4x^3$

Thus, $df/dx = 50t^{49} \times (4x^3)$

$= 50(x^4 – 1)^{49} \times (4x^3)$

$= 200\ x^3(x^4 – 1)^{49}$

Example 2:

Find the derivative of $f(x) = e^{\sin(2x)}$

Solution:

Given,

$f(x) = e^{\sin(2x)}$

Let $t = g(x) = \sin 2x$ and $u(t) = e^t$

According to chain rule,

$df/dx = (du/dt) \times (dt/dx)$

Here,

$du/dt = d/dt\ (e^t) = e^t$

$dt/dx = d/dx\ g(x)$

$= d/dx (\sin 2x)$

$= 2 \cos 2x$

Thus, $df/dx = e^t \times 2 \cos 2x$

$= e^{\sin(2x)} \times 2 \cos 2x$

$= 2 \cos(2x) e^{\sin(2x)}$